

13. G. Emad, W.M. Roth, Contradictions in the practices of training for and assessment of competency: A case study from the maritime domain, *Education and Training* 50 (2008) 260-272.
14. S. Nazir, R. Totaro, S. Brambilla, S. Colombo, D. Manca, Virtual Reality and Augmented-Virtual Reality as Tools to Train Industrial Operators, *Computer Aided Chemical Engineering* 30 (2012) 1398-1401.
15. Johnson, J. D. (2017). Interprofessional care teams: the perils of fads and fashions. *International Journal of Healthcare Management*, 10(2), 127-134.
16. D. Manca, S. Nazir, F. Lucernoni, S. Colombo, Performance Indicators for the Assessment of Industrial Operators, *Computer Aided Chemical Engineering* 30 (2012) 1422-1426.
17. P.W. Johnston, E. Fioratou, R. Flin, Non-technical skills in histopathology: Definition and discussion, *Histopathology* 59 (2011) 359-367.
18. D. Manca, S. Nazir, S. Colombo, A. Kluge, Procedure for automated assessment of industrial operators, *Chemical Engineering Transactions* 36 (2014) 391-396.
19. Sussman, E. D., Bishop, H., Madnick, B., & Walter, R. (1985). Driver inattention and highway safety.

УДК 629.5.051

doi.org/10.33298/2226-8553.2024.3.41.17

Abramov G.S., Plotnikov V.I.

MATHEMATICAL MODELING OF THE PROCESSES OF INCREASING THE RELIABILITY OF THE NAVIGATION COMPLEX THROUGH REDUNDANCY

The purpose of the article is research and mathematical modeling of how the reliability of the navigation complex can be increased by adding redundant elements to the system. The presented article examines some issues of determining the stability of technical systems, first of all, from the point of view of their reliability (probability of failure-free operation). A number of relevant problems related to increasing the reliability of systems by adding redundant elements are considered. The assessment of various schemes for redundancy of system elements was carried out and their mathematical modeling was carried out. For systems consisting of nonuniformly reliable elements, the validity of replacing their reliabilities with an averaged value has been investigated. It is shown that such a replacement gives somewhat inflated system reliability values, and makes sense only with a small dispersion of reliability values (probabilities of failure-free operation) of system elements. At high dispersion values, the relative error of the system reliability calculation increases sharply ($>10\%$). An example of greater or less deep redundancy of the least reliable elements of the system is also presented, which significantly (more than twice) increases the reliability of the system as a whole. The directions of further research are defined, such as the use of the concept of full probability and Bayesian (posterior) probabilities of the relevant hypotheses.

Keywords: redundancy. Reliability. probability of trouble-free operation. navigation complex. modeling.

Description of the problem. Currently, the technological aspect of a ship's navigation system consists of a network consisting of components, subsystems, nodes and human-machine interfaces. The bridge team uses this equipment to perform maritime tasks such as monitoring, forecasting and decision-making during vessel handling, ensuring safe navigation throughout the voyage. Sensors, radio and communication equipment, data sources, data processing tools, and visualization tools are part of the components and subsystems. Their performance depends on configuration and human control.

Thus, the ship navigation systems used can be classified either as technical systems (without human participation) or as socio-technical systems (with human participation). The specific type of the navigation system ultimately determines which criteria should be used to assess the presence of stability. If the safe and efficient navigation of a vessel is a priority, the technical system and the crew must be considered as a single unit working together with the changing environment, taking into account both the technology and the capabilities of the crew.

A purely technological perspective assesses technical robustness (reliability) by measuring how well technical systems can provide critical navigational information with the required efficiency. When considering socio-technical factors, it is important to recognize the role of humans as they configure, communicate, control and make decisions for the navigation system. They also serve as the body that initiates and makes adjustments in response to emerging threats. In this context, stability is defined as the successful management of a vessel during a voyage with minimal safety risks [1].

It is widely recognized that human operators play a critical role in ensuring the safety of navigation by compensating for failures or deficiencies in the technical system, such as incorrect or late collision alarms. On the other hand, the current technical systems are not yet stable enough to compensate for the low productivity of the human operator. The grounding of the Costa Concordia occurred due to several contributing factors, [2] but none of the technical systems could adapt quickly enough to counteract the lack of situational awareness and poor decision-making by human operators on the bridge crew.

The concept of applying resilience principles to improve navigational safety and improve shipping has been around for some time [3–12] and has been used in various areas such as improving ship traffic services, investigating ship incidents, implementing Safety Approach II [13] for the maritime industry and improving awareness of traffic conditions for safe navigation. All papers emphasize that focusing on individual factors alone is not enough to achieve resilience. Furthermore, regardless of whether the emphasis is on methodology or human factors, it is important to depict ship navigation as a series of processes, taking into account dependencies and interactions. An example is the maritime perspective model in [1], which offers a generalized process model for navigational decision-making. The main difficulty lies in balancing the knowledge and experience of the initial parameters (vessel profiling) and the current situation (situation assessment) for making navigational decisions in uncertain situations and their development.

Analysis of the latest research and publications. The principles of resilience, as stated in [14–16], define abstract potential sources of stability in engineering systems. The success of a potential source of resilience depends on how it is integrated into the system, taking into account the approach used and its impact on other measures implemented to increase the resilience of the system.

Jackson [14;15] noted that over 40 resilience principles have been developed for engineering systems over the past few decades. He proposed to divide the entire array of principles into 14 main principles, and also to include such subsets of principles (sub-principles) as margin, automated function or regrouping sub-principles. The implementation of a specific principle of resilience helps to create or expand a certain ability of the system in relation to the desired attribute of the system [14; 16]. Jackson presented four characteristics as the goals of resilience: the ability to withstand a threat (capacity), the ability to adapt to a threat (flexibility), maintain functionality in the presence of a threat (tolerance), and remain united when faced with a threat (cohesion).

Woods [17] classified a number of sustainability principles based on four different resilience concepts targeting different objectives. The initial idea (reliability as resilience) involves the system's ability to operate effectively under typical and somewhat degraded circumstances. The second idea emphasizes effective and efficient recovery after traumatic and destructive events (resilient recovery). A third idea considers resilience in situations where a system is operating near the limits of, or beyond, its capacity, or is facing unexpected and new threats (resilience as opposed to fragility). "Resilient adaptability" refers to the management of functionality and performance in a changing and interconnected world that includes assumptions, user requirements, framework conditions such as economic, environmental and legal aspects, and the diversity of their relationships and interactions.

Sterbenz and his colleagues [18–20] developed a number of recommendations for creating powerful information networks and communication systems. In these publications, fault tolerance principles are

called enablers and include both general approaches (such as redundancy and diversity) and application-specific approaches (such as context awareness and transparency).

Purpose and objectives of the publication. The purpose of this work is to determine a method of increasing the reliability of navigation systems using various redundancy schemes for its elements (aggregates, blocks, modules). The main tasks: 1) assessment and modeling of the reliability of various schemes for redundant system elements; 2) assessment of the reliability of a system with equally reliable component elements; 3) assessment of the reliability of a system with unequally reliable constituent elements.

Summary of the main material. In order to determine the level of reliability of navigation equipment by the method of expert evaluation, a survey of 22 captains and senior assistant captains with work experience of 5 to 8 years was conducted. On the basis of the received answers, diagrams were formed, which show how often each component of the navigation equipment of a sea vessel fails (Fig. 1) and whether it would be appropriate to add redundancy for a specific component (Fig. 2).



Figure 1 – Frequency of failures per component

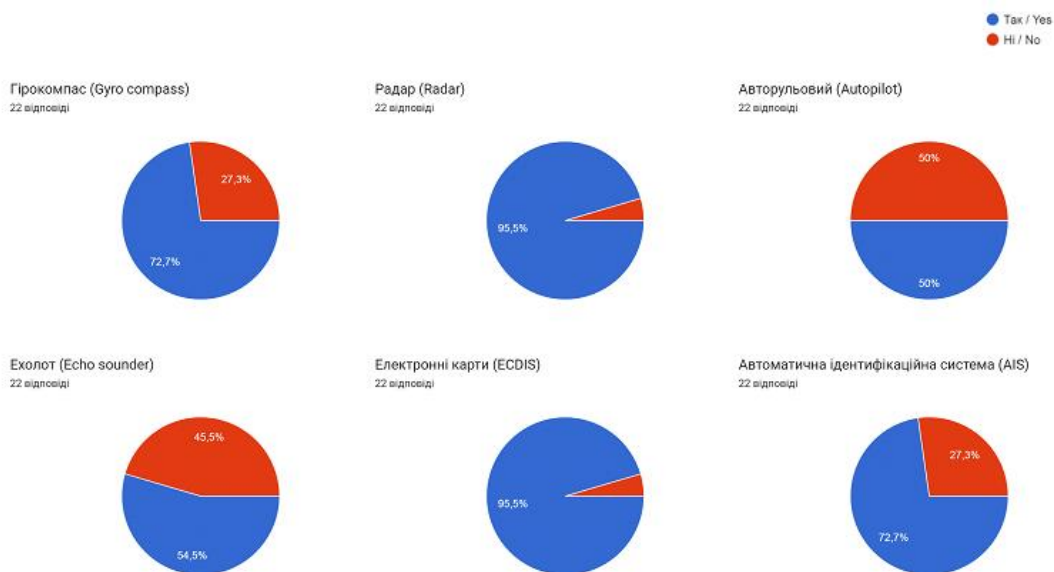


Figure 2 – Importance of redundancy per component

As can be seen above, some components of navigation equipment have insufficient reliability and therefore need improvements.

One of the most common methods of increasing equipment reliability is greater or lesser redundancy of its elements. Redundancy is especially relevant in case of insufficient primary reliability of the elements of the equipment complex.

If there are no fundamental technological possibilities to improve the quality and, thereby, the reliability of the elemental base of the equipment, then redundant components are almost the only means of increasing the operational reliability of the equipment complex as a whole.

Probability theory methods [21;22] make it possible to theoretically calculate the reliability of redundant systems and evaluate the effectiveness of various schemes for redundant elements.

We will consider a number of relevant, in our opinion, problems related to the reliability of the equipment of the navigation complex from the point of view of redundancy of its constituent elements.

1. Evaluation and modeling of the reliability of various schemes for duplicating system elements.

Let the complex (system) of some navigation equipment contain a certain number of n elements, the probability of their trouble-free operation is p . The efficient operation of the equipment requires failure-free operation of all its components.

In order to increase the reliability of the equipment, the system is duplicated, for which n more elements are allocated,

Let's consider two possible duplication schemes in this setting:

a) duplication of each element

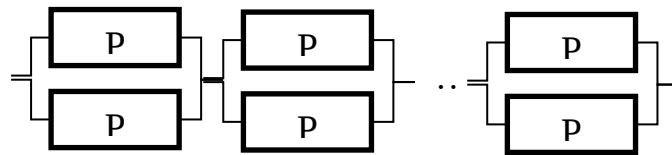


Figure 3 – Scheme of duplication of each element

b) duplication of the entire system

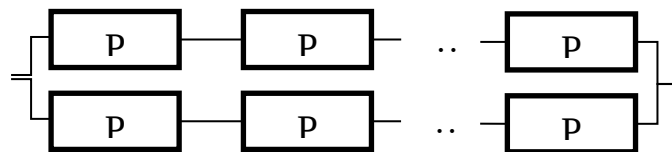


Figure 4 – Scheme of duplication of the entire system

We will carry out appropriate calculations and numerical simulations to find out which duplication scheme gives higher operational reliability.

Reliability P_a (as the probability of failure-free operation), which is provided by duplication according to scheme "a"), has the following form:

$$P_a = [1 - (1 - p)^2]^n \quad (1)$$

The corresponding expression of P_b for scheme "b" has the form:

$$P_b = 1 - (1 - p^n)^2 \quad (2)$$

By comparing (1) and (2), we will show that $P_a > P_b$.

We reformulate expressions (1) and (2):

$$P_a = [1 - (1 - p)^2]^n = p^n(2 - p)^n \quad (3)$$

$$P_b = 1 - (1 - p^n)^2 = p^n(2 - p^n) \quad (4)$$

Let's prove the inequality:

$$(2 - p)^n > 2 - p^n \quad (5)$$

Considering that $q = 1 - p$, inequality (5) will have the following form:

$$(2 - 1 + q)^n > 2 - (1 - q)^n, \text{ або} \quad (6)$$

$$(1 + q)^n + (1 - q)^n > 2 \quad (7)$$

Applying the binomial formula, we get:

$$\begin{aligned} (1 + q)^n + (1 - q)^n &= 1 + nq + \frac{n(n-1)}{2!}q^2 + \frac{n(n-1)(n-2)}{3!}q^3 + \dots \\ \dots + 1 - nq + \frac{n(n-1)}{2!}q^2 - \frac{n(n-1)(n-2)}{3!}q^3 + \dots &= \\ &= 2 + n(n-1)q^2 + 0 \left(\frac{n(n-1)(n-2)(n-3)}{12}q^4 \right) > 2 \end{aligned} \quad (8)$$

Thus, $(2 - p)^n > 2 - p^n$, and this means that scheme "a" is better, more reliable than scheme "b", i.e. duplication of each element of the system provides higher reliability than duplication of the system as a whole.

The analysis of inequality (8) shows that the advantages of scheme "a" will be more obvious when the primary reliability of the elements is insufficient, that is, when the probability values of p are far enough away from unity.

On the contrary, when p approaches unity, the difference between schemes "a" and "b" is equalized and the reliability (probability of failure-free operation) of both schemes asymptotically approaches unity.

We will demonstrate the obtained results by carrying out numerical simulations according to equations (3) and (4) for different values of n and in a wide range of variation of p values (from 0.7 to 0.99).

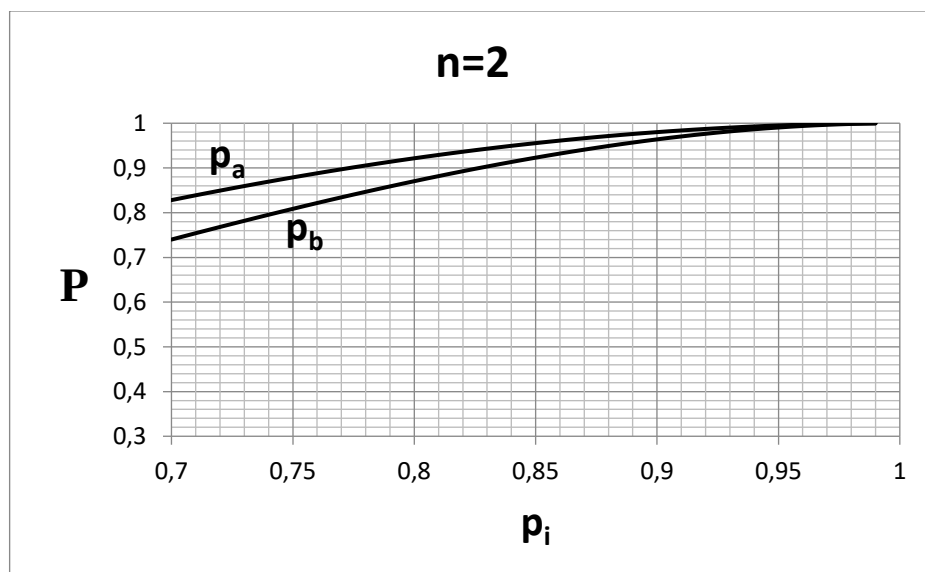


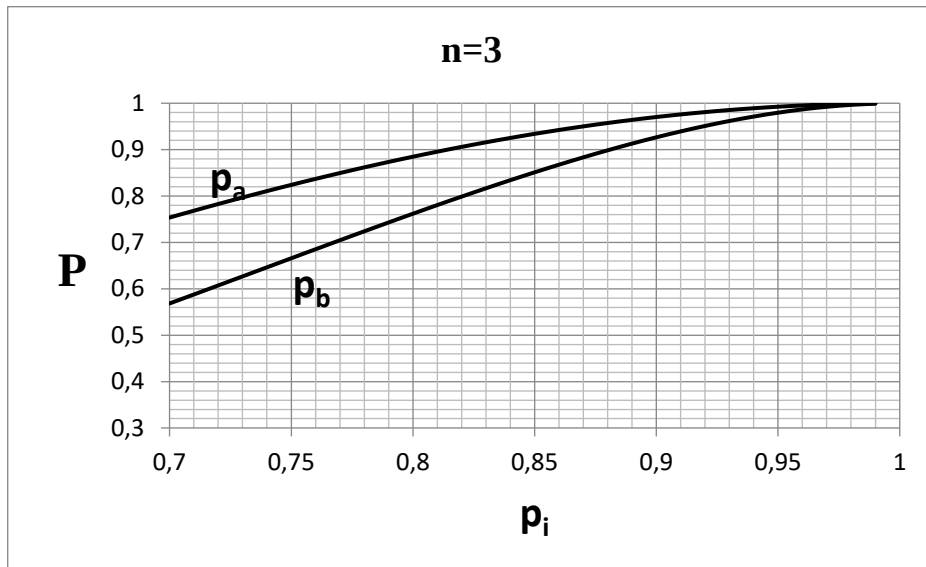
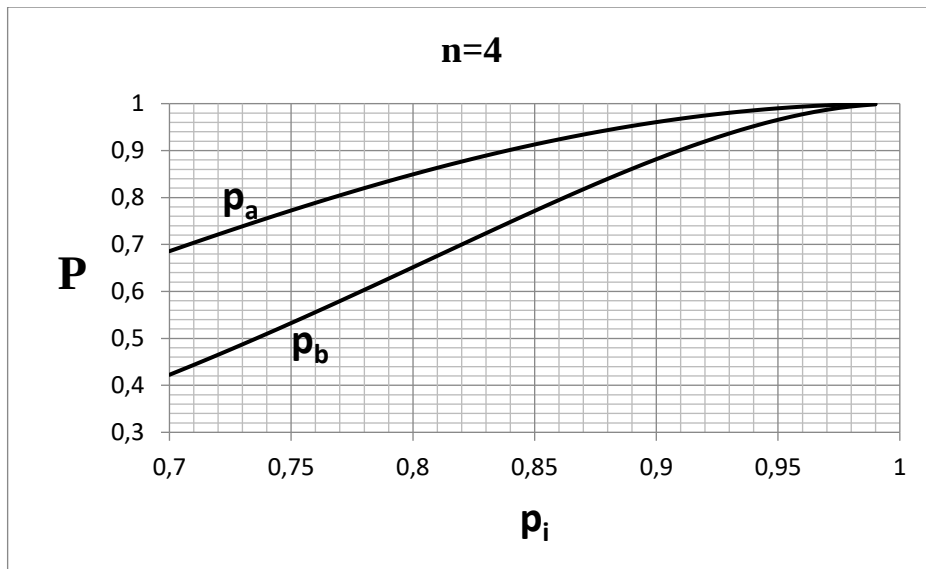
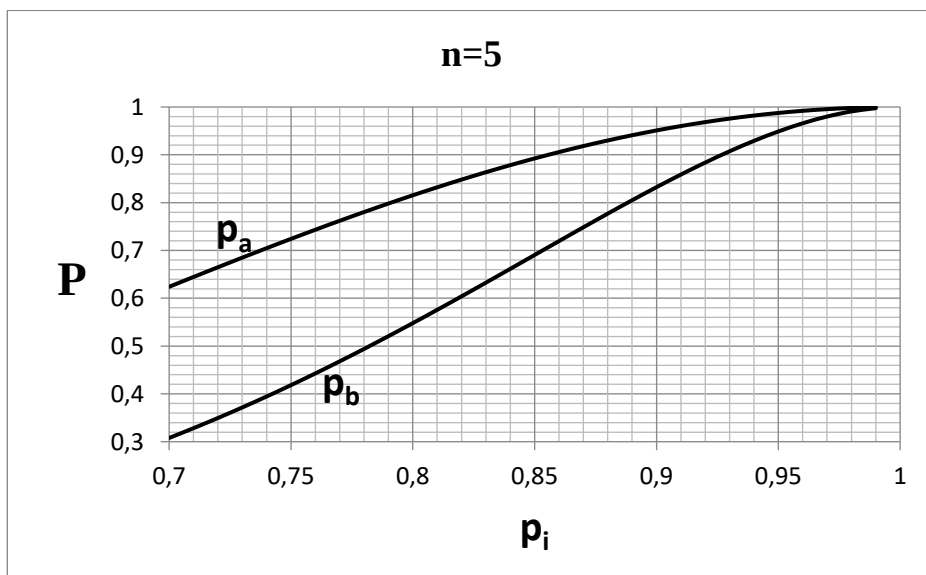
Figure 5 – Simulation results with $n=2$ Figure 6 – Simulation results with $n=3$ Figure 7 – Simulation results with $n=4$ 

Figure 8 – Simulation results with $n=5$

In fig. 5–8 present the simulation results that confirm the conclusions derived from the analysis of inequality (8), but an unobvious and unexpected result is the fact that as n increases, scheme "b" gives significantly lower reliability than the primary reliability of p elements, if the latter is not exceeds the values of 0.85–0.9. That is, if the values of p are not high enough, then duplication according to scheme "b" will not only not increase the reliability of the system, but, on the contrary, will worsen it.

The type of reliability curves according to scheme "b" in fig. 5–8 suggests the existence of inflection points. As you know, to determine them, it is necessary to find the second derivative of expression (4) and equalize it to zero.

$$P'_6 = 2np^{n-1}(1 - p^n) \quad (9)$$

$$P''_6 = 2n[(n-1)p^{n-2}(1 - p^n) - np^{2(n-1)}] \quad (10)$$

Equating the last expression to zero, we have:

$$p^{2n}(2n-1) - (n-1)p^n = 0 \quad (11)$$

Thereby:

$$p^n(2n-1) = n-1 \quad (12)$$

That is, the expression for the inflection point has the form:

$$P = \left(\frac{n-1}{2n-1} \right)^{\frac{1}{n}} \quad (13)$$

Thereby: $n=2$; $p = \sqrt{\frac{1}{3}} \approx 0,577$; $n=3$; $p = \left(\frac{2}{5}\right)^{\frac{1}{3}} \approx 0,737$; $n=4$; $p = \left(\frac{3}{7}\right)^{\frac{1}{4}} \approx 0,809$; $n=5$; $p = \left(\frac{4}{9}\right)^{\frac{1}{5}} \approx 0,850$

Comparison of the received coordinates of the inflection points with the curves for P_b in fig. 5–8 shows that the above calculation is correct. Knowing the coordinates of the inflection point for scheme "b" can serve as a certain criterion, starting from which the reliability of this scheme will asymptotically approach unity.

It seems necessary to consider two problems about the reliability of a technical system with n elements under the condition of equal reliability of its component elements and in the absence of such.

2. Evaluation of the reliability of a system with equally reliable constituent elements.

The diagram of a technical system of equally reliable elements, the operation of each of which is necessary for the successful operation of the system as a whole, is presented in fig. 9:



Figure 9 – A system of equally reliable component elements

If the technical system consists of n equally reliable elements, the failure of each of which means the failure of the system as a whole, then the overall reliability of the system (probability of failure-free operation) P :

$$P = p^n, \quad (14)$$

whence, the reliability p of each element to ensure the overall reliability P must be equal:

$$p = \sqrt[n]{P} \quad (15)$$

It is also possible to find a limit for the number of elements n with reliability p to ensure the specified reliability of the system P :

$$n = \text{int}\left[\frac{\ln P}{\ln p}\right] \quad (16)$$

where $\text{int}[x]$ - integer part of x .

Let's illustrate the given dependence:

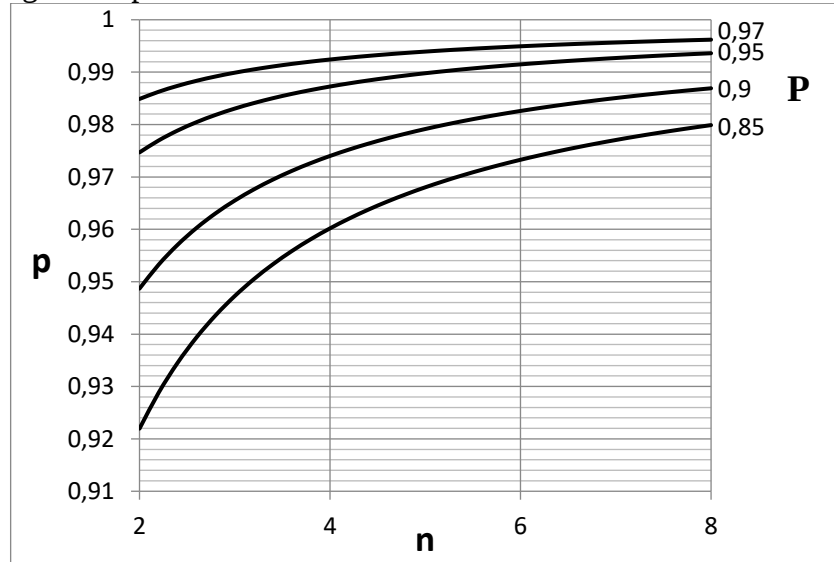


Figure 10 – Dependence (15) of the required reliability of elements p on the general of the (given) reliability of the system P and the number of elements in it n

It can be seen that with an increase in the given reliability of the system P and the number of elements n in it, an increasingly high reliability of the constituent elements of the system p is required.

3. Reliability assessment of a system with unequally reliable component elements.

If the system consists of non-uniformly reliable elements (which probably happens most often in practice) and it is quite difficult to estimate the probability of failure-free operation of each element separately, then they are often replaced by an approximated, average value:

$$\bar{p} = \frac{1}{n} \sum_{i=1}^n p_i \quad (17)$$

Let's evaluate how valid such a replacement is and how the probabilities of the system's failure-free operation, which are calculated based on the true values of p_i and their average value, will differ.

If the probabilities p_i of failure-free operation of each element were known, then under the condition of independence of the failure of each element, the exact value of the probability of failure-free operation of the system as a whole $P = \prod_{i=1}^n p_i$.

The approximate value of $\bar{P}$, calculated from the average probability $\bar{p}$, will have the form:

$$\bar{P} = \left(\frac{1}{n} \sum_{i=1}^n p_i\right)^n \quad (18)$$

Thus, it is necessary to compare the values $\prod_{i=1}^n p_i$ and $\left(\frac{1}{n} \sum_{i=1}^n p_i\right)^n$.

It is known that the geometric mean of positive (not equal to each other) values is less than their arithmetic mean:

$$\sqrt[n]{\prod_{i=1}^n p_i} < \frac{1}{n} \sum_{i=1}^n p_i \quad (19)$$

whence it follows that:

$$\prod_{i=1}^n p_i < \left[\frac{1}{n} \sum_{i=1}^n p_i \right]^n \quad (20)$$

i.e., $P < \bar{P}$, which means that the estimate of the probability of failure-free operation of the system $\bar{P}$ – based on the average values for p_i will always be slightly overestimated compared to the true one.

From a priori considerations, it seems that this difference should depend on the dispersion of the array of p_i values.

We will perform numerical experiments on the example of the following array:

p_i : 0.55; 0.6; 0.65; 0.7; **0.75**; 0.8; 0.85; 0.9; 0.95.

We will sequentially examine sets of 3, 5, 7, and 9 elements with an average value of $\bar{p} = \mathbf{0.75}$.

We will calculate the variances according to known formulas:

$$s^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 = \frac{1}{n} \sum_{i=1}^n x_i^2 - \bar{x}^2 \quad (21)$$

Accordingly, the mean square deviation (standard error):

$$s = \sqrt{s^2} \quad (22)$$

Let's also calculate the coefficient of variation $\gamma = \frac{s}{\bar{x}} \cdot 100\%$, as the relative value of the error and the relative error δ between the values of $\bar{P}$ and P :

$$\delta = \frac{\bar{P} - P}{P} \cdot 100\% \quad (23)$$

The obtained results are shown in Table 1.

We will give an example of calculating the first row of the table $n = 3$: 0,7; **0,75**; 0,8.

$$P = 0,7 \cdot 0,75 \cdot 0,8 = 0,42; \bar{P} = 0,75^3 = 0,422$$

$$s^2 = \frac{1}{3} (0,49 + 0,5625 + 0,64) - 0,5625 = 0,00167$$

$$s = 0,041; \gamma = \frac{0,041}{0,75} \cdot 100\% = 5,47\%$$

$$\delta = \frac{0,422 - 0,42}{0,42} \cdot 100\% = 0,48\%$$

Table 1 – Values of P , $\bar{P}$ and errors s , γ and δ for sets of 3, 5, 7 and 9 elements

n	P	$\bar{P}$	s	$\gamma, \%$	$\delta, \%$
3	0,42	0,422	0,041	5,47	0,48
5	0,232	0,237	0,071	9,47	2,16
7	0,1253	0,1335	0,1	13,3	6,54
9	0,0655	0,0751	0,1292	17,23	14,7

It can be seen that with the increase in the dispersion of p_i values, the relative error δ increases sharply, its values exceed 10%, which calls into question the legality of replacing the array p_i with their average value.

4. An example of a scheme for duplicating a system of unequally reliable elements.

If the system consists of non-uniformly reliable elements, it is necessary to consider the problem of duplicating elements whose reliability is not high enough, because they limit the reliability of the system in general. The effectiveness of duplicating the least reliable elements of the system can be shown in the following example.

Let the technical system consist of five elements whose reliability (probability of failure-free operation) is equal to: $p_1 = 0,85$; $p_2 = 0,75$; $p_3 = p_4 = 0,95$; $p_5 = 0,75$. It can be seen that the 1st, 2nd and 5th elements need to be duplicated as insufficiently reliable, and the 2nd and 5th elements should have, apparently, three times duplication (Fig. 11).

Without duplication, the reliability of the system is:

$$\bar{P} = [1 - (1 - p_1)] \cdot [1 - (1 - p_2)] \cdot [1 - (1 - p_3)] \cdot [1 - (1 - p_4)] \cdot [1 - (1 - p_5)] \\ = 0,85 \cdot 0,75 \cdot 0,95 \cdot 0,95 \cdot 0,75 = 0,418$$

By the way, if reliability is determined by the average value $\bar{p} = 0,85$, then it will be, as should be expected, a little higher:

$$\bar{P} = \left(\frac{1}{n} \sum_{i=1}^n p_i \right)^n = 0,85^5 = 0,444$$

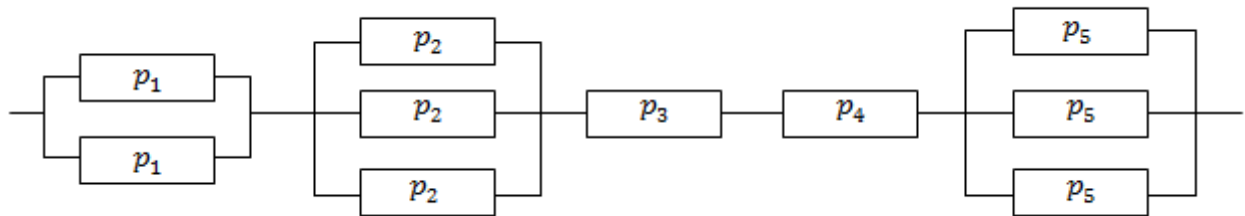


Figure 11 – Scheme of the duplicated system

The reliability of the duplicated system (Fig. 11) is:

$$P = [1 - (1 - p_1)^2] \cdot [1 - (1 - p_2)^3] \cdot p_3 \cdot p_4 \cdot [1 - (1 - p_5)^3] \\ = 0,9775 \cdot 0,984 \cdot 0,95 \cdot 0,95 \cdot 0,984 = 0,854$$

As you can see, the reliability of the system, the elements of which are duplicated accordingly, increases more than twice and exceeds 85%.

CONCLUSIONS

In this work, the effectiveness of various schemes for duplicating elements of the navigation complex systems and their mathematical modeling was evaluated. The obtained results made it possible to draw a number of conclusions regarding the probability of failure-free operation of the system when using one or another redundancy scheme.

If the system consists of equally reliable elements, duplicating each element is a more effective method of increasing reliability than duplicating the entire system. Moreover, if the value of the primary reliability of the elements is equal to 0.85 or lower, then the duplication of the entire system with each increase in the number of duplicate elements will significantly reduce the reliability of the system. At the same time, increasing the reliability of the system by duplicating each element with insufficient primary reliability of the elements will be more significant at values of the reliability of the elements below 0.85.

If the system consists of nonuniformly reliable elements, then if it is impossible to estimate the probability of failure-free operation of each element separately, the replacement of their values with an averaged value turns out to be unreliable (with a significant dispersion of their values), since with a high dispersion of the values of the probability of failure-free operation of system elements, the value of the relative error increases sharply, which may exceed 10%.

Duplication of the least reliable elements of a system consisting of unequally reliable elements can increase the reliability of the system more than twice, provided that it is done taking into account the value of the probability of failure-free operation of each element.

Абрамов Г.С., Плотніков В.І.

МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ПРОЦЕСІВ ПІДВИЩЕННЯ НАДІЙНОСТІ НАВІГАЦІЙНОГО КОМПЛЕКСУ ШЛЯХОМ ДУБЛЮВАННЯ.

У статті розглядаються проблеми оцінювання стійкості технічних систем, насамперед зосереджуючись на їх надійності в умовах безвідмовної роботи. Метою цього дослідження є дослідження та математичне моделювання підвищення надійності за допомогою різних схем дублювання в компонентах навігаційної системи.

Обговорювалися різноманітні актуальні питання щодо підвищення надійності систем шляхом дублювання їх компонентів. Було оцінено різні методи копіювання компонентів системи та розроблено їх математичне представлення. Проведено дослідження доцільності заміни надійностей елементів у системах з різним рівнем надійності середнім значенням. Було встановлено, що дублювання кожного елемента в системі рівно надійних елементів є більш ефективним способом підвищення надійності, ніж дублювання всієї системи. Крім того, якщо первинне значення надійності елементів становить 0,85 або менше, дублювання всієї системи з кожним додаванням повторюваних елементів значно знизить надійність системи. Підвищення надійності системи шляхом дублювання елементів з низькою первинною надійністю стає більш важливим, коли надійність елемента нижче 0,85. Також було продемонстровано, що така заміна призводить до трохи вищих оцінок надійності системи, і вона є логічною лише тоді, коли існує мінімальна варіація значень надійності елементів системи. При великих значеннях дисперсії відносна похибка розрахунку надійності системи різко зростає більш ніж на 10%. Показано приклад суттєвого дублювання найменш надійних компонентів системи, що призвело до більш ніж подвійного підвищення загальної надійності системи. Майбутні дослідження повинні зосередитися на використанні концепції повної ймовірності та Байєсівської ймовірності для відповідних гіпотез.

Ключові слова: дублювання; надійність; ймовірність безвідмовної роботи; навігаційний комплекс; моделювання.

REFERENCES

1. Wahlström, M., Forster, D., Karvonen, A., Puustinen, R., Saariluoma, P. (2019). Perspective-Taking in Anticipatory Maritime Navigation – Implications for Developing Autonomous Ships. *In Proceedings of the 18th International Conference on Computer and IT Applications in the Maritime Industries*, Tullamore, Ireland, 25–27 March 2019; 191–200.
2. Wreathall, J. (2011). Monitoring—A critical ability in Resilience Engineering. *In Resilience Engineering in Practice: A Guidebook*; Hollnagel, E., Paries, J., Woods, D.D., Wreathall, J., Eds.; Ashgate Pub Co.: Surrey, UK.
3. Praetorius, G., Hollnagel, E. (2014). Control and Resilience Within the Maritime Traffic Management Domain. *J. Cogn. Eng. Decis. Mak.*, 8, 303–317.
4. Praetorius, G. (2014). Vessel Traffic Service (VTS): A Maritime Information Service or Traffic Control System? Understanding Everyday Performance and Resilience in a Socio-Technical System under Change. *Ph.D. Thesis*, Chalmers University of Technology, Gothenburg, Sweden.
5. Schröder-Hinrichs, J.-U., Praetorius, G., Graziano, A., Kataria, A., Baldauf, M. (2015). Introducing the Concept of Resilience into Maritime Safety. *In Proceedings of the 6th Resilience Engineering Association Symposium*, Lisboa, Portugal, 22–25 June 2015.

6. Smith, D., Veitch, B., Khan, F., Taylor, R. (2013). Using the FRAM to Understand Arctic Ship Navigation: Assessing Work Processes During the Exxon Valdez Grounding. *TransNav Int. J. Mar. Navig. Saf. Sea Transp.*, 12, 447–457.
7. Wahlström, M. (2019). Resilience on the seven seas: perspective-taking in anticipatory ship navigation. In *Proceedings of the 8th REA Symposium Embracing Resilience: Scaling Up and Speeding Up*, Kalmar, Sweden, 24–27 June 2019.
8. van Westrenen F., Praetorius G. (2014). Maritime traffic management: a need for central coordination? *Cognition, Technology & Work*, 16(1), 59–70.
9. Aven, T., Andersen, H.B., Cox, T., Droguett, E.L., Greenberg, M., Guikema, S., Kröger, W., Renn, O., Zio, E., 2015. Risk Analysis Foundations. *Soc. Risk Anal.*
10. Ayyub, B.M., 2015. Practical Resilience Metrics for Planning, Design, and Decision Making. ASCE-ASME J. Risk Uncertain. Eng. Syst. Part Civ. Eng. 1. doi:<http://dx.doi.org/10.1061/AJRUA6.0000826> - doi:10.1061/AJRUA6.0000826
11. Ayyub, B.M., 2014. Systems resilience for multihazard environments: definition, metrics, and valuation for decision making. *Risk Anal. Off. Publ. Soc. Risk Anal.* 34, 340–355. doi:10.1111/risa.12093 - doi:10.1111/risa.12093
12. Smith, D., Veitch, B., Khan, F., Taylor, R., 2015. An Accident Model for Arctic Shipping, in: *Proceedings of the ASME 2015 34th International Conference on Ocean, Offshore and Arctic Engineering*. Presented at the OMAE 2015, St. John's, NL, Canada
13. Hollnagel, E. (2011). RAG—The resilience analysis grid. In *Resilience Engineering in Practice: A Guidebook*; Hollnagel, E., Paries, J., Woods, D.D., Wreathall, J., Eds.; Ashgate Pub Co.: Surrey, UK.
14. Jackson, S., Ferris, T.L.J. (2013). Resilience Principles for Engineered Systems. *Syst. Eng.*, 16, 152–164.
15. Jackson, S., Ferris, T.L.J. (2016). Designing Resilient Systems. In *Proceedings of the NATO Advanced Research Workshop on Resilience-Based Approaches to Critical Infrastructures Safeguarding*, Azores, Portugal, 26–29 June 2016; 121–144.
16. Madni, A.M., Jackson, S. (2009). Towards a Conceptual Framework for Resilience Engineering. *IEEE Syst. J.*, 3, 181–191.
17. Woods, D.D. (2015). Four concepts for resilience and the implication for the future resilience. *Reliab. Eng. Syst. Saf.*, 141, 5–9.
18. Sterbenz, J.P.G., Hutchison, D., Cetinkaya, E.K., Jabbar, A., Rohrer, J., Schöller, M., Smith, P. (2010). Resilience and survivability in communication networks: Strategies, principles, and survey of disciplines. *Comput. Netw.*, 54, 1245–1265.
19. Sterbenz, J.P.G., Hutchison, D., Cetinkaya, E.K., Jabbar, A., Rohrer, J., Schöller, M., Smith, P. (2014). Redundancy, diversity, and connectivity to achieve multilevel network resilience, survivability, and disruption tolerance. *Telecommun. Syst.*, 56, 17–31.
20. Cheng, Y., Todd Gardner, M., Li, J., May, R., Medhi, D., & Sterbenz, J. P. G. (2014). Optimised heuristics for a geodiverse routing protocol. In *The 10th IEEE/IFIP international conference on Design of Reliable Communication Networks (DRCN)*, Ghent, Belgium.
21. Feller, V. (1984). *Vvedenie v teoriyu veroyatnostej i eyo prilozheniya*. (Vols. 1-2). M.: Mir.
22. Heche, F.E. (2019). *Teoriia ymovirnostei i matematychna statystyka*. Uzhhorod: PP «AUTDOR-ShARK».

ЛІТЕРАТУРА

1. Wahlström M., Forster D., Karvonen A., Puustinen R., Saariluoma P. Perspective-Taking in Anticipatory Maritime Navigation – Implications for Developing Autonomous Ships. In *Proceedings of the 18th International Conference on Computer and IT Applications in the Maritime Industries*, Tullamore, Ireland, 25–27 March 2019. P. 191–200.
2. Wreathall J. Monitoring—A critical ability in Resilience Engineering. In *Resilience Engineering in Practice: A Guidebook*; Hollnagel E., Paries J., Woods D.D., Wreathall J., Eds.; Ashgate Pub Co.: Surrey, UK. 2011.

3. Praetorius G., Hollnagel E. Control and Resilience Within the Maritime Traffic Management Domain. *J. Cogn. Eng. Decis. Mak.* 2014. Vol. 8, P. 303–317.
4. Praetorius G. Vessel Traffic Service (VTS): A Maritime Information Service or Traffic Control System? Understanding Everyday Performance and Resilience in a Socio-Technical System under Change. *Ph.D. Thesis*, Chalmers University of Technology, Gothenburg, Sweden. 2014.
5. Schröder-Hinrichs J.-U., Praetorius G., Graziano A., Kataria A., Baldauf M. Introducing the Concept of Resilience into Maritime Safety. *In Proceedings of the 6th Resilience Engineering Association Symposium*, Lisboa, Portugal, 22–25 June 2015.
6. Smith D., Veitch B., Khan F., Taylor R. Using the FRAM to Understand Arctic Ship Navigation: Assessing Work Processes During the Exxon Valdez Grounding. *TransNav Int. J. Mar. Navig. Saf. Sea Transp.* 2013. Vol. 12, P. 447–457.
7. Wahlström M. Resilience on the seven seas: perspective-taking in anticipatory ship navigation. *In Proceedings of the 8th REA Symposium Embracing Resilience: Scaling Up and Speeding Up*, Kalmar, Sweden, 24–27 June 2019.
8. van Westrenen F., Praetorius G. (2014). Maritime traffic management: a need for central coordination? *Cognition, Technology & Work*, 16(1), 59-70.
9. Aven, T., Andersen, H.B., Cox, T., Droguett, E.L., Greenberg, M., Guikema, S., Kröger, W., Renn, O., Zio, E., 2015. Risk Analysis Foundations. *Soc. Risk Anal.*
10. Ayyub, B.M., 2015. Practical Resilience Metrics for Planning, Design, and Decision Making. *ASCE-ASME J. Risk Uncertain. Eng. Syst. Part Civ. Eng.* 1. doi:<http://dx.doi.org/10.1061/AJRUA6.0000826> - doi:10.1061/AJRUA6.0000826
11. Ayyub, B.M., 2014. Systems resilience for multihazard environments: definition, metrics, and valuation for decision making. *Risk Anal. Off. Publ. Soc. Risk Anal.* 34, 340–355. doi:10.1111/risa.12093 - doi:10.1111/risa.12093
12. Smith, D., Veitch, B., Khan, F., Taylor, R., 2015. An Accident Model for Arctic Shipping, in: *Proceedings of the ASME 2015 34th International Conference on Ocean, Offshore and Arctic Engineering*. Presented at the OMAE 2015, St. John's, NL, Canada
13. Hollnagel E. RAG—The resilience analysis grid. *In Resilience Engineering in Practice: A Guidebook*; Hollnagel E., Paries J., Woods D.D., Wreathall J., Eds.; Ashgate Pub Co.: Surrey, UK. 2011.
14. Jackson S., Ferris T.L.J. Resilience Principles for Engineered Systems. *Syst. Eng.* 2013. Vol. 16, p. 152–164.
15. Jackson S., Ferris T.L.J. Designing Resilient Systems. *In Proceedings of the NATO Advanced Research Workshop on Resilience-Based Approaches to Critical Infrastructures Safeguarding*, Azores, Portugal, 26–29 June 2016. P. 121–144.
16. Madni A.M., Jackson S. Towards a Conceptual Framework for Resilience Engineering. *IEEE Syst. J.* 2009. Vol. 3, P. 181–191.
17. Woods D.D. Four concepts for resilience and the implication for the future resilience. *Reliab. Eng. Syst. Saf.* 2015. Vol. 141, P. 5–9.
18. Sterbenz J.P.G., Hutchison D., Cetinkaya E.K., Jabbar A., Rohrer J., Schöller M., Smith P. Resilience and survivability in communication networks: Strategies, principles, and survey of disciplines. *Comput. Netw.* 2010. Vol. 54, P. 1245–1265.
19. Sterbenz J.P.G., Hutchison D., Cetinkaya E.K., Jabbar A., Rohrer J., Schöller M., Smith P. Redundancy, diversity, and connectivity to achieve multilevel network resilience, survivability, and disruption tolerance. *Telecommun. Syst.* 2014. Vol. 56, P. 17–31.
20. Cheng, Y., Todd Gardner, M., Li, J., May, R., Medhi, D., & Sterbenz, J. P. G. (2014). Optimised heuristics for a geodiverse routing protocol. *In The 10th IEEE/IFIP international conference on Design of Reliable Communication Networks (DRCN)*, Ghent, Belgium.
21. Феллер В. Введение в теорию вероятностей и её приложения. В 2-х томах. М.: Мир, 1984. Т.1: 528 с., Т.2: 738 с.
22. Гече Ф.Е. Теорія ймовірностей і математична статистика: навч. посібн. Ужгород: ПП «АУТДОР-ШАРК», 2019. 235 с.